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Entropy generation minimization of free convection film condensation on an elliptical cylinder

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Abstract

This paper aims to perform thermodynamic analysis of saturated vapor flowing slowly onto and condensing on an elliptical cylinder. This analysis provides us how the geometric parameter-ellipticity affects entropy generation during film-wise condensation heat transfer process. The results observe that local condensate film thickness decreases with an increase in ellipticity of a cylinder. From the first law point of view, the local heat transfer coefficient enhances as ellipticity increases. Meanwhile, from the second law point of view, entropy generation increases with increasing the value of ellipticity. We derive an expression for entropy generation, which accounts for the combined action of the specified irreversibilities. The result demonstrates that thermal irreversibility dominates over film flow friction irreversibility. Finally, an expression of minimizing entropy generation in laminar film condensation heat transfer is obtained.

Keywords: Free convection; Film-wise; Condensation; Thermodynamic second law; Elliptical cylinder

1. Introduction

Conservation of useful energy depends on the design of efficient thermodynamic heat-transfer processes, i.e. minimization of entropy generation due to heat transfer and viscous dissipation. Entropy generation in thermal engineering systems destroys system available energy and reduces its efficiency.

As for enhancement of condensation heat transfer, several researches, such as Yang and Hsu [1] and Yang and Chen [2], Ali and McDonald [3], Karimi [4], and Memory et al. [5] confirmed that cylinders, fins, or extended surfaces of elliptical profiles with major axes aligned with gravity are superior to those of circular profiles. It is a fact that heat transfer enhancement is achieved, however the increase in heat transfer rate is known to augment friction factor due to pumping power.

Bejan [6] pioneered the method of entropy generation minimization in heat and mass transfer analysis. He found thermodynamics optimums of the ratio of film coefficient to pumping power and the dimensionless temperature difference with constant mass flow rate and heat transfer rate per unit length. He devised concrete methods for minimizing entropy generation in engineering equipment for heat transfer. He conducted EGM analysis on ducts with constant heat flux for flat plates; cylinders in cross flow. Sahin [7] investigated the effect of temperature-dependent viscosity on the entropy generation rate as well as the ratio of pumping power to heat transfer.

Adeyinka and Naterer [8] investigated the physical significance of entropy generation in plate film condensation. Lin et al. [9] first performed the second-law analysis on saturated vapor flowing through and condensing inside horizontal cooling tubes. They noted that in a tube case, an optimum Reynolds number exists at which the entropy generates at a minimum rate. Dung and Yang [10] presented the entropy generation minimization method to optimize a saturated vapor flowing slowly onto and condensing on an isothermal horizontal tube. Their results for the optimizing entropy generation and plate size are expressed in terms of a duty parameter. In addition, they observed that entropy generation provides a useful parameter in the optimization of a two-phase system.

Entropy generation is associated with thermodynamic irreversibility which is common in all types of heat transfer

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Nomenclature

a	semi-major axis of ellipse
b	semi-minor axis of ellipse
Bo	Bond number, $(\rho - \rho_v)ga^2/\sigma$
Br	Brinkman number, $\mu u_0^2/k\Delta T$
C_p	specific heat capacity of condensate
D_e	equivalent diameter of elliptical cylinder
e	ellipticity of ellipse
g	acceleration due to gravity
h	condensing heat transfer coefficient at angle ϕ
h'_{fg}	latent heat of condensation corrected for condensate subcooling
Ja	Jakob number, $C_p(T_{sat} - T_w)/h'_{fg}$
k	thermal conductivity of condensate
$\dot{m}$	condensate mass flow rate per unit length of elliptical cylinder
N_F	film flow friction irreversibility
N_H	heat transfer irreversibility
N_S	the entropy generation number
Nu	local Nusselt number, hD_e/k
S'''_{gen}	the volumetric entropy generation rate, defined in Eq. (16)
S'''_o	characteristic transfer rate
Ra	Rayleigh number, $(\rho - \rho_v)\rho g\, Pr D_e^3/\mu^2$
T_{sat}	saturation temperature of vapor
T_w	wall temperature
u	velocity component in x direction
v	velocity component in y direction

Greek symbols

δ	thickness of condensate film
δ^*	dimensionless thickness of condensate film, defined in Eq. (14)
θ	angle measured from top of the cylinder
μ	absolute viscosity of condensate
ρ	density of condensate
ρ_v	density of vapor
σ	surface tension coefficient in the film
ϕ	angle between the tangent to cylinder surface and the normal to direction of gravity
φ	the irreversibility distribution ratio, defined in Eq. (26)
Θ	dimensionless temperature difference, $\Delta T/T$

Subscripts

opt	optimal
sat	saturation
v	vapor
w	cylinder wall

Superscripts

$*$	indicates dimensionless

processes. Film condensation belongs to phase-change heat transfer, but little literature regarding its second-law analysis is investigated. The second law analysis of the film condensation outside cylinders still remains an unsettled question so far. More recently, we first conducted a study [11] on the local entropy generation rate of laminar free convection film condensation on an elliptical cylinder. That paper investigated how the geometric parameter-ellipticity affects local entropy-generation rate during film-wise condensation heat transfer process.

Currently, the present study will focus on the minimization of total entropy generation number to give an idea of optimal design on free convection film condensation outside an elliptical cylinder. We derive an expression for the entropy generation number, which accounts for the combined action of *finite-temperature difference heat transfer irreversibility and film flow friction irreversibility*. Basically, this study makes good engineering sense to focus on the irreversibility of film condensation heat transfer and try to understand the function of the entropy generation mechanism. This investigation on the entropy generation minimization will thus help us achieve the complete thermodynamic analysis, including first and second law, on laminar film-wise condensation outside an elliptical cylinder.

2. Thermal analysis

Consider a horizontal elliptical cylinder with major axis "2a" in the gravitational direction and minor axis "2b", situated in a *slowly flowing* pure vapor which is at its saturated temperature T_{sat}. Moreover, the wall temperature T_w is considered to be uniform and much lower than the vapor saturation temperature T_{sat}. Thus, condensation occurs on the wall and a continuous film of the liquid runs downward over the cylinder under the influence of gravity.

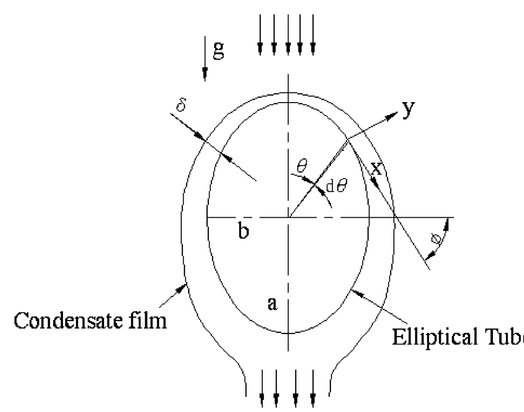

Fig. 1. Physical model and coordinate system for condensate film flow on an elliptical surface.

Fig. 1 illustrates schematically a physical model and coordinate system where the curvilinear coordinates (x, y) are aligned along an elliptical cylinder surface and its normal. The assumptions employed in the formulation of the problem are:

(1) The condensate film flow is laminar *and steady*.
(2) The inertia effect of *the condensate film flow* is neglected.
(3) The condensate film thickness is much smaller than the curvature diameter.
(4) Viscous dissipation in the interface is ignored.
(5) Compared with the transversal conduction *within the condensate film*, the axial conduction is negligible.

According to above assumptions, the condensate film governed equations are written respectively.

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{1}$$

Momentum equation

$$\mu\frac{\partial^2 u}{\partial y^2}=-(\rho-\rho_v)g\big[\sin\phi+Bo(\phi)\big] \tag{2}$$

Energy equation

$$k\frac{\partial^2 T}{\partial y^2}=0 \tag{3}$$

subject to the following boundary conditions:

$$y=0,\quad u=0,\quad T=T_{\mathrm{w}} \tag{4}$$

$$y=\delta,\quad \frac{\partial u}{\partial y}=0,\quad T=T_{\mathrm{sat}} \tag{5}$$

On account of varying radius of surface curvature, the surface tension forces can be derived here, as expressed in Yang and Chen [2]:

$$Bo(\phi)=\pm\frac{1}{Bo}\frac{3e^2\sigma}{2a^2}\left(\frac{1-e^2\sin^2\phi}{1-e^2}\right)^2\sin(2\phi) \tag{6}$$

Integrating Eqs. (2) and (3) directly with the boundary conditions gives the following formula of the film velocity "u" and temperature "T" profile, respectively.

$$u=\frac{\rho-\rho_v}{\mu}g\delta^2\big[\sin\phi+Bo(\phi)\big]\left[\frac{y}{\delta}-\frac{1}{2}\left(\frac{y}{\delta}\right)^2\right] \tag{7}$$

$$T=\Delta T\frac{y}{\delta}+T_{\mathrm{w}} \tag{8}$$

By assuming a reference velocity,

$$u_0=\frac{(\rho-\rho_v)gD_{\mathrm{e}}^2}{2\mu} \tag{9}$$

Eq. (7) becomes:

$$u(y)=u_0\big(2y\delta-y^2\big)\big[\sin(\phi)+Bo(\phi)\big]/D_{\mathrm{e}}^2 \tag{10}$$

Using Eq. (7), we can obtain the mass flow rate per unit length

$$\dot m=\rho(\rho-\rho_{\mathrm{v}})\frac{g\delta^3}{3\mu}\big[\sin\phi+Bo(\phi)\big] \tag{11}$$

Since the temperature distribution in the condensate layer may be assumed linear in the Nusselt–Rohsenow condensation theory, one has

$$\frac{\mathrm{d}\dot m}{\mathrm{d}x}=\frac{k\Delta T}{h'_{\mathrm{fg}}\delta} \tag{12}$$

where, $h'_{\mathrm{fg}}=h_{\mathrm{fg}}(1+0.68C_{\mathrm{p}}\Delta T/h_{\mathrm{fg}})$. In order to derive the local film thickness δ at the circumferential arc length x in terms of ϕ, one can substitute Eq. (11) into Eq. (12) and obtain

$$\frac{\rho(\rho-\rho_v)}{3\mu}gh'_{\mathrm{fg}}\frac{\mathrm{d}}{\mathrm{d}x}\big(1-e^2\sin^2\phi\big)^{3/2}\frac{\mathrm{d}}{\mathrm{d}\phi}\big\{\delta^3\big[\sin\phi+Bo(\phi)\big]\big\}=\frac{k\Delta T}{\delta} \tag{13}$$

Using the transformation method from x to ϕ, *as introduced in Yang and Chen* [2], we can derive dimensionless local condensate liquid film thickness as.

$$\delta^*=\delta\left[\frac{D_{\mathrm{e}}k\mu\Delta T}{gh'_{\mathrm{fg}}\rho(\rho-\rho_v)}\right]^{-1/4}=\big[\sin\phi+Bo(\phi)\big]^{-1/3}\times\frac{\big\{2\big(1-e^2\big)\int_0^\phi\frac{[\sin\phi+Bo(\phi)]^{1/3}}{(1-e^2\sin^2\phi)^{3/2}}\,\mathrm{d}\phi\big\}^{1/4}}{\big\{\frac{1}{\pi}\int_0^\pi\big[(1-e^2)/\sqrt{(1-e^2\sin^2\phi)^3}\,\mathrm{d}\phi\big]\big\}^{1/4}} \tag{14}$$

As in Nusselt [12] theory, interpreting the result of model is straightforward by employing the usual idea of a local heat transfer coefficient as follows:

$$Nu=\frac{hD_{\mathrm{e}}}{k}=\frac{[Ra/Ja]^{1/4}}{\delta^*} \tag{15}$$

where,

$$Ra\equiv\frac{\rho(\rho-\rho_v)g\,Pr\,D_{\mathrm{e}}^3}{\mu^2}$$

$$Ja\equiv\frac{C_p\Delta T}{h'_{\mathrm{fg}}}$$

According to Bejan [13], together with the fifth item of above-mentioned assumptions, the entropy generation rate for convection heat transfer can be written as

$$\dot S'''_{\mathrm{gen}}=\frac{k}{T^2}\left(\frac{\partial T}{\partial y}\right)^2+\frac{\mu}{T}\left(\frac{\partial u}{\partial y}\right)^2 \tag{16}$$

On the right-hand side of Eq. (16), the first term and the second term represents the entropy generation due to heat transfer and due to film flow friction, respectively. Substituting Eqs. (8) and (10) into Eq. (16) and assuming $\Delta T\,y/\delta\ll T_{\mathrm{w}}$ yield

$$\dot S''_{\mathrm{gen}}=\frac{k}{T_{\mathrm{w}}^2}\left(\frac{\Delta T}{\delta}\right)^2+\frac{\mu}{T_{\mathrm{w}}}\left\{\frac{u_0(2\delta-2y)[\sin(\phi)+Bo(\phi)]}{D_{\mathrm{e}}^2}\right\}^2 \tag{17}$$

Integrating Eq. (17) with respect to y from zero to δ yields

$$S'_{\mathrm{gen}}=\frac{k}{T_{\mathrm{w}}^2}\left(\frac{\Delta T}{\delta}\right)^2\delta+\frac{4\mu}{3T_{\mathrm{w}}}\left\{\frac{u_0[\sin(\phi)+Bo(\phi)]}{D_{\mathrm{e}}^2}\right\}^2\delta^3 \tag{18}$$

Next, integrating Eq. (18) over the entire streamline length, from $\phi=0$ to π gives

$$S_{\mathrm{gen}}=\frac{k(\Delta T)^2}{2T_{\mathrm{w}}^2}(Ra/Ja)^{1/4}I_{\mathrm{r}}+\frac{2u_0^2\mu}{3T_{\mathrm{w}}}(Ra/Ja)^{-3/4}I_{\mathrm{d}} \tag{19}$$

where,

$$I_r = \int_0^{\pi} \frac{1}{\delta^*}\,d\phi \quad \text{and} \quad I_d = \int_0^{\pi} (\delta^*)^3\big[\sin(\phi) + Bo(\phi)\big]^2\,d\phi \tag{20}$$

k, T_w and μ denote thermal conductivity, wall temperature, and dynamic viscosity, respectively. Entropy generation number (N_S'') is the ratio of the volumetric entropy generation rate (S_{gen}) to a characteristics transfer rate (S_o).

$$N_S = \frac{S_{gen}}{S_o} \tag{21}$$

where,

$$S_o = \frac{k(\Delta T)^2}{T_w^2} \tag{22}$$

Further, by introducing the following dimensionless parameters

$$Br = \mu\frac{u_0^2}{k\Delta T} \tag{23}$$

$$\Theta = \frac{\Delta T}{T_W} \tag{24}$$

the entropy generation number can be expressed as:

$$N_S = \frac{1}{2}(Ra/Ja)^{1/4} I_r + \frac{2}{3}\frac{Br}{\Theta}(Ra/Ja)^{-3/4} I_d = N_H + N_F \tag{25}$$

To understand which of the condensate flow friction irreversibility (N_F), or heat transfer irreversibility (N_H) dominates, we introduce a criterion known as the irreversibility distribution ratio in the following equation:

$$\varphi = \frac{N_F}{N_H} \tag{26}$$

Setting $\frac{\partial N_S}{\partial (Ra/Ja)} = 0$, we find the following optimum that minimizes value of N_S

$$(Ra/Ja)_{opt} = 4\frac{I_d}{I_r}\frac{Br}{\Theta} \tag{27}$$

Inserting Eq. (27) into Eq. (20) gives an expression of minimizing entropy generation as follows:

$$(N_S)_{opt} = (Ra/Ja)_{opt}^{\frac{1}{4}}\frac{2}{3}I_r \tag{28}$$

The ratio of the actual entropy generation to the minimized entropy generation represents N_S^*, which is determined to be

$$\begin{aligned} N_S^* &= \frac{N_S}{(N_S)_{opt}} \\ &= \frac{\frac{1}{2}(Ra/Ja)^{\frac{1}{4}} I_r}{\frac{2}{3}(Ra/Ja)_{opt}^{\frac{1}{4}} I_r} + \frac{\frac{2}{3}(Ra/Ja)^{\frac{-3}{4}}\frac{Br}{\Theta} I_d}{\frac{2}{3}(Ra/Ja)_{opt}^{\frac{1}{4}} I_r} \\ &= \frac{3}{4}\left[\frac{Ra/Ja}{(Ra/Ja)_{opt}}\right]^{\frac{1}{4}} + \frac{1}{4}\left[\frac{(Ra/Ja)_{opt}}{Ra/Ja}\right]^{\frac{3}{4}} \end{aligned} \tag{29}$$

3. Results and discussion

Fig. 2 indicates the variation of dimensionless entropy generation numbers N_S with Ra/Ja under the surface tension effects for various ellipticities. Firstly, from the first law point of view, we also confirm that the mean heat transfer coefficients enhance with value of ellipticity as stated in Yang and Chen [2] study. Secondly, from the second law point of view, because dimensionless entropy generation numbers increase with mean heat transfer coefficients, N_S is proportional to Ra/Ja. Therefore, dimensionless entropy generation numbers increases with an increase in the ellipticity and Ra/Ja. Thirdly, entropy generation number is nearly unaffected by surface tension forces at small ellipticity like $e \leqslant 0.7$, but somewhat influenced at large ellipticity for whole perimeters. Accordingly, the effect of surface tension on the entropy generation number is significant at a larger ellipticity.

Fig. 3 shows that the total dimensionless entropy generation numbers N_S is almost induced by the heat transfer irreversibil-

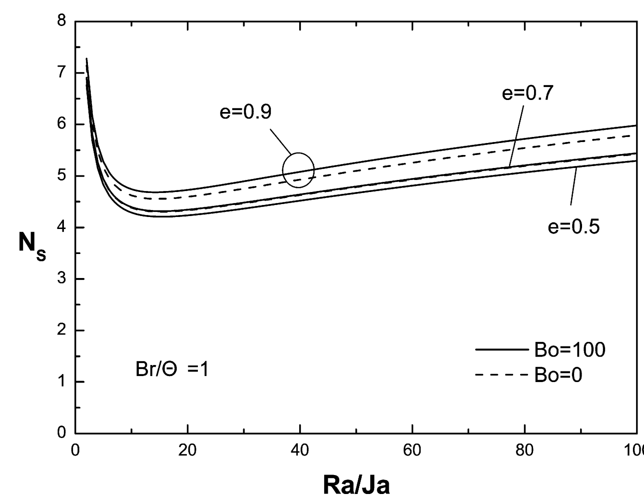

Fig. 2. The variation of dimensionless entropy generation numbers N_S with surface tension effect and ellipticities versus Ra/Ja.

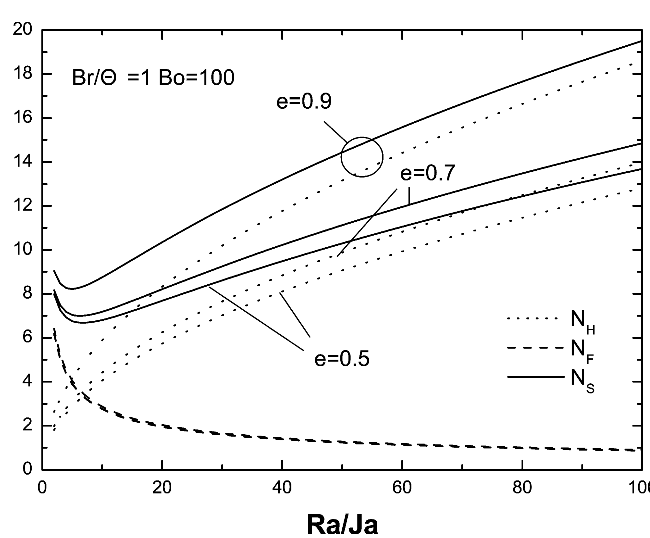

Fig. 3. Dimensionless entropy generation number versus Ra/Ja for every kind of ellipticities.

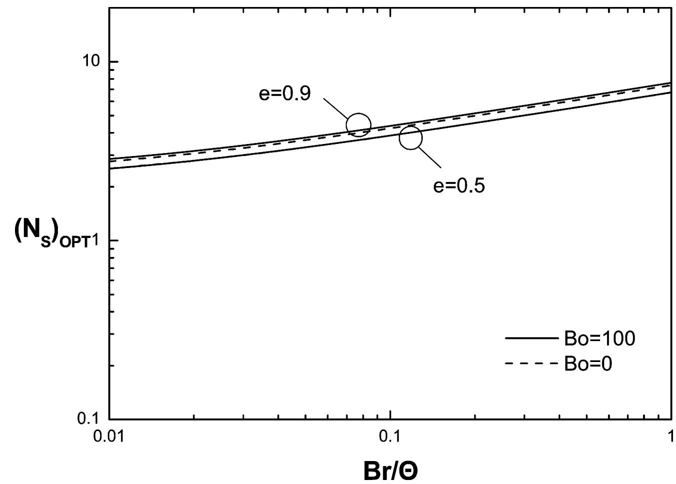

Fig. 4. Minimum entropy generation rate versus Br/Θ.

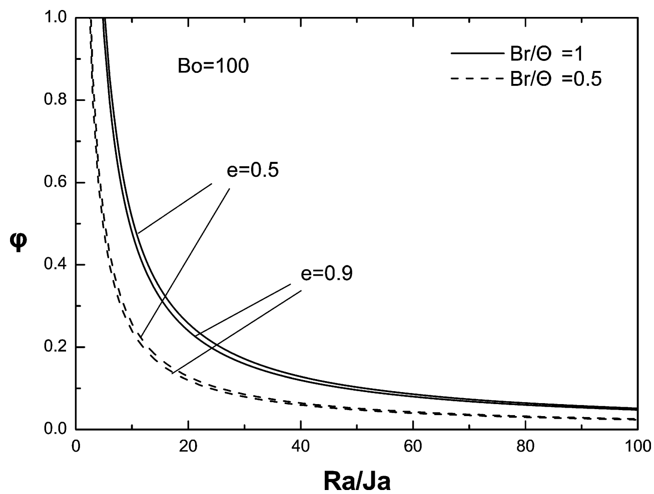

Fig. 5. The irreversibility distribution ratio versus Ra/Ja.

ity, N_H i.e. the irreversibility due to heat transfer across the finite film temperature difference is much more than that due to the film flow friction. Eq. (25) reads that heat transfer generation number N_H varies as the square root of Ra/Ja, while the film flow friction generation number N_F varies as the inverse square root of Ra/Ja. Apparently from Fig. 3, heat transfer generation number is much more than the film flow friction generation number when $Ra/Ja > 5$.

Fig. 4 shows minimum entropy generation rate versus Br/Θ. From Eq. (27), one may clearly see that the optimal value of Ra/Ja varies as Br/Θ. We thus know relationship of minimum entropy generation rate and Br/Θ in Eq. (28) i.e. $(N_S)_{opt}$ varies as square root of Br/Θ. As indicated in Fig. 4, the effect of surface tension on minimum entropy generation can be ignored.

When $\varphi < 1$ in Fig. 5, the heat transfer irreversibility dominates over the flow friction irreversibility. This may account to the finite temperature difference heat transfer across the condensate film thickness. Although, there exists a gravity-induced film flow friction irreversibility within the condensate film, this viscous drag contribution to the entropy generation rate declines across the film. Therefore, irreversibility distribution ratio φ decreases with Ra/Ja, but increases with Br/Θ. In

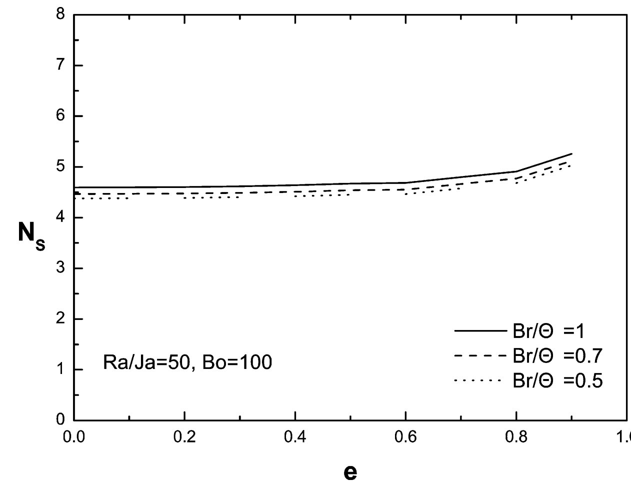

Fig. 6. Minimum entropy generation rate versus ellipticities.

general, for the case of free-convection film condensation, the entropy-generation rate due to gravity-induced film flow friction is usually small.

Considering the effect of geometrical parameters-ellipticity on the irreversibility distribution ratio, we may observe that irreversibility distribution ratio decreases with an increase in the ellipticity. This means that the heat transfer irreversibility dominated is more significant for the lower ellipticity.

Next, minimum entropy generation rate versus ellipticities in Fig. 6 demonstrates that total dimensionless entropy generation numbers increase with Br/Θ and ellipticities, similar to the mean heat transfer coefficient trend as described in Yang and Hsu [1].

4. Conclusions

This is the first approach using the entropy generation minimization to investigate free convection film-wise condensation on an elliptical cylinder. The result obtained only applies to the very slow or quiescent vapor condensed outside horizontal elliptical cylinders, and to very long elliptical cylinders, with negligible interfacial vapor shear drag. The foregoing results can be summarized as follows:

1. The entropy generation number was found to be a function of the group Rayleigh, Brinkman numbers and geometrical parameters-ellipticity.
2. The effect of group parameters on entropy generation number and the irreversibility distribution ratio was examined, respectively.
3. We can find that heat transfer generation number N_H and dimensionless entropy generation numbers N_S increase with the values of ellipticities and Ra/Ja, but film flow friction generation number N_F declines to nil as Ra/Ja increases.
4. Because of assuming $\Delta T y/\delta \ll T_w$, the actual entropy generation rate is less than that of the present calculated result.
5. The optimal design can be achieved by analyzing entropy generation in film condensation on elliptical cylinders;

however, the practical ellipticity is limited to 0.9 owing to manufacturing availability.

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